Synthesis of All-Optical Microwave Filters Using Mach–Zehnder Lattices

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***Abstract*— A synthesis algorithm which generates the set of coupling ratios required to produce a desired time-domain window response using an all-optical Mach–Zehnder lattice network is presented. The analysis assumes incoherent interference of the lightwaves within the structure. The window coefficients are dictated by the coupling ratios of the couplers forming the lattice, leading to a simple structure comprising only passive components. Since its impulse response (IR) is finite, the filter will be stable, and the algorithm is capable of generating a wide range of responses in terms of their extinction ratios and passbands. The theory has been validated by experiment.**

***Index Terms*—Microwave filters, optical-fiber delay lines.**

I. INTRODUCTION

THE ANALYSIS of all-optical structures for microwave signal processing has been the subject of much research over the past decade [1]–[3]. However, it is only recently that the *synthesis* of such structures has been considered [4]–[7]. Synthesis of infinite impulse-response (IIR) filters using positive coefficients was considered in [4]. The limitations of positive coefficient IIR design were overcome in [5], [6] by employing differential optical detection. Nevertheless, these methods still lead to complex IIR structures—the stability of which cannot be guaranteed. Finite impulse-response (FIR) structures on the other hand are always stable. Synthesis techniques for FIR optical delay-line filters were presented in [7], using cascaded fixed coupling ratio $j \times j$ $(j \geq 2)$ couplers, and in [8], which relied upon optical amplifiers to generate the coefficients. The disadvantage of such an approach is that the use of optical amplifiers significantly increases the cost and complexity of the filters. In contrast, the synthesis technique presented in this paper allows FIR structures to be designed which incorporate only passive components. The resulting all-optical microwave filters are, therefore, robust, economical, and simple to manufacture. By ensuring incoherent optical interference, the output impulses of the filter are restricted to positive-only values. This then allows the application of data windows (which have all-positive coefficients [9], [10]) to the synthesis problem.

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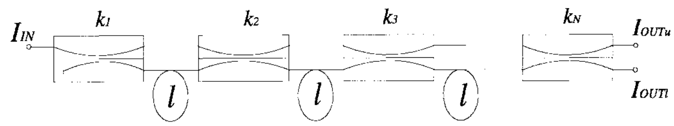

Fig. 1. The Mach–Zehnder lattice topology with unit delay length l.

The structure used by the design algorithm is the simplest of the all-optical FIR topologies, the feed-forward, or Mach–Zehnder lattice [1]. This is presented along with a new and rigorous analysis of the optical-intensity impulse response (IR), and its relationship with the coupling ratios in the structure. Based on the results of this analysis, a novel technique for synthesizing symmetrical time-domain IR's is then described, and a set of the algorithm's output values is included. Finally, the results of the microwave characterization of such a structure are shown and discussed.

II. THE MACH–ZEHNDER LATTICE

A. Analysis

A single optical Mach–Zehnder section consists of two directional couplers separated by two unequal optical paths. A Mach–Zehnder lattice is produced when several such sections are cascaded [1], as in Fig. 1. In the analysis which follows, the couplers are optical-fiber directional couplers whose behavior is modeled using [1, eq. (3)]. The unit time delay—the difference between the two path lengths—introduces a factor of z^{-1} to the path transmittance, and is realized by a length of optical fiber. Assuming the input port to be the upper arm of the first coupler, we shall now consider the general form of the IR of this structure. For a lattice containing N couplers, there are, in total, $2^{(N-1)}$ possible forward paths between the input port and each of the output ports. Since each of these paths must pass once through each of the couplers, with the ith coupler introducing a transmittance factor of either k_i when the path couples across the coupler, or $(1-k_i)$ when the path travels directly through it, the magnitude of each of the path transmittances will be a product of N factors, such that

$$t_i = \prod_{l=1}^{N} C_l, \qquad \text{for } i = 1 \text{ to } 2^{(N-1)}. \tag{1}$$

Here, t_i is the ith forward-path graph transmittance, and C_l will be equal to either k_l or $(1-k_l)$, depending on the route taken by the path. These path transmittances contribute to the output impulses in such a way that the ith output impulse is the sum of the transmittances of all those forward paths introducing $(i-1)$ unit delays. For an N coupler structure there are N such output impulses. The upper and lower arm impulse IR's are, therefore, both of the form

$$H(z) = \sum_{i=1}^{N} h(i) z^{(1-i)} \tag{2}$$

where $h(i)$ is the magnitude of the ith impulse of the IR. The binomial nature of the structure means that the number of paths contributing to each of the output impulses follows the coefficients of Pascal's triangle, so that in the N coupler structure, the ith impulse will be the sum of n products, where

$$n = \frac{N!}{(i-1)!(N-i+1)!}. \tag{3}$$

The coefficients are relatively straightforward to generate, but the resulting impulses are clearly nonlinear with respect to the values of k_i, and as the order of the filter response increases so does the complexity of the synthesis problem. However, by a qualitative consideration of the general IR of a structure incorporating N optical-directional couplers, several conclusions can be drawn, which are the theoretical basis of the algorithm to be presented.

Firstly, by conservation of energy

$$\sum_{i=1}^{N} (h_u(i) + h_l(i)) = 1. \tag{4}$$

Secondly, we consider the effect on any single-path transmittance of altering the way in which the path crosses the first of the couplers, from direct to cross-coupling or *vice versa*. If this alteration is imposed on one of the paths there are two effects. Firstly, all the delays that were included in the original path are now omitted, and conversely, all the delays that were omitted are included. Secondly, the path will now terminate on the other output port. This observation can be formalized using the expression

$$t_u(i) = \frac{C_1}{(1-C_1)} t_l\left(2^{(N-1)} + 1 - i\right) \tag{5}$$

where $t_u(i)$ is any path which terminates at the upper output port and contributes to output impulse i, $t_l(i)$ is the lower output port equivalent, and C_1 is the first factor of the magnitude of t_u, i.e., either k_1 or $(1-k_1)$.

By the same token, the effect of altering the way in which the path traverses the final coupler in the lattice section gives

$$t_u(i) = \frac{C_N}{(1-C_N)} t_l(i) \tag{6}$$

where C_N is the final factor of the magnitude of t_u, i.e., either k_N or $(1-k_N)$.

B. Synthesis

In the synthesis technique presented in this paper, the first coupling coefficient is given a value of 0.5, which has two important effects on the IR. Firstly, it divides all the optical power available from the input impulses equally between the upper and lower arms of the structure. Neglecting losses, once this is the case it will remain so throughout the lattice, because of the reciprocal nature of the coupler's behavior, giving

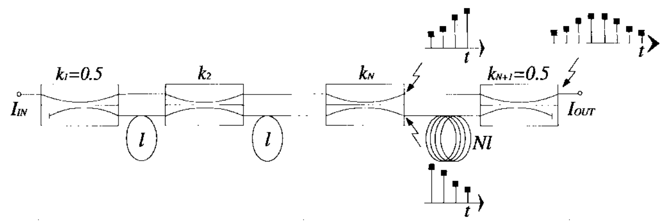

Fig. 2. The Mach–Zehnder lattice topology adapted to produce a symmetrical time-domain response.

$$\sum_{i=1}^{N} h_l(i) = \sum_{i=1}^{N} h_u(i) = 0.5. \tag{7}$$

Paths that differ only in the way they traverse the first coupler will now have equal transmittances, and so, from (5) we can deduce

$$h_u(i) = h_l(N+1-i) \tag{8}$$

which means that the IR in the top arm is a reversed version of that of the bottom arm, when the impulses are analyzed at the output of any of the couplers. This, in turn, means that the IR at the lower port of any of the couplers in the main Mach–Zehnder chain is both reversed and delayed by the unit time delay in comparison with that in the upper arm.

Finally, the ratio of the first output impulse to the last is now solely determined by the final coupling coefficient, i.e., from (6) and (8)

$$h_u(1) = \frac{(1-k_N)}{k_N} h_u(N). \tag{9}$$

It has been shown that when $k_1 = 0.5$, the IR of the lower output is the reverse of the upper output. If the lower IR is delayed by an appropriate amount and then combined with the upper IR, the structure can, therefore, be used to generate a symmetrical IR (see Fig. 2). As windows are symmetrical, they can be generated by using an N coupler Mach–Zehnder chain to produce the first half of the window at the upper output, and adding a final Mach–Zehnder section which is unbalanced by the delay $N\tau$, and has a final coupling ratio of 0.5. Exploiting the symmetry of the IR in this way ensures that an IR of $(2(N-1))$ impulses can be generated by a chain of N couplers, the first and last of which have coupling coefficients of 0.5. This clearly reduces the potential cost and size of the structure. Knowing half of the window IR dictates the value of the last coupling coefficient in the main chain [from (9)] and knowledge of this value in turn enables the desired IR at the input to the final coupler to be deduced. This then becomes the output of the previous coupler which can be used to find the value of the $(N-1)$th coupling ratio. The process is repeated, gradually regressing to the first coupler in the chain, at which point all the values of the coupling coefficients k_i, for i from 1 to N, are known.

The design procedure for a window response is, therefore, as follows.

Step 1) Calculate the required unit delay time from the design frequency. In some cases it may be useful to exploit the periodic nature of the intensity response in the frequency domain.

Step 2) Evaluate the window coefficients bearing in mind that the number of impulses of the IR will be equal to $2(x-1)$ for a structure composed of a total of x couplers. These values must then be normalized so that their sum is 1.

Step 3) Take the first half of this window to be the design IR giving the values of $h(i)$ for $i = 1$ to N.

Step 4) Calculate the coupling coefficient of the final coupler using

$$k_N = \left(1 + \frac{h(1)}{h(N)}\right)^{-1}. \qquad (10)$$

Step 5) Use this value of k_N to calculate the values of the $(N-1)$ impulses at the input of the final coupler, using where $h'(i)$ is the new generated IR

$$h'(i) = \frac{(1-k_N)h(i) - k_N h(N+1-i)}{(1-2k_N)}, \quad \text{for } i = 2 \text{ to } N-1. \qquad (11)$$

Step 6) Repeat steps 4 and 5 using the new IR to work out the preceding coupling coefficient, and so on, until all the coefficients are known.

Step 7) Complete the structure by adding a delay of N times the unit delay length to the output of the lower arm of the structure, and combining this with the output of the upper arm, using a coupler with $k = 0.5$.

There are some IR's that defy generation in this way. There are two main difficulties which cause the process to fail to generate a realizable final structure. One occurs when the first and last impulses in the IR being generated are equal. This necessitates a final coupling coefficient in the main chain of 0.5, which means that as a consequence of (5) and (6)

$$h_u(i) = h_l(i) = h_u(n+1-i) = h_l(n+1-i) \qquad (12)$$

so that the upper-arm IR is both the same and the reverse of the lower-arm IR. This can clearly only be the case if the upper- and lower-arm IR's are themselves symmetrical.

The other problem arises when

$$k_N < \frac{1}{2} \quad \text{and} \quad \frac{h(i)}{h(N+1-i)} < \frac{k_N}{(1-k_N)} \qquad (13)$$

because this implies negative coupling coefficients. The general form of windows guarantees that neither of these situations arise when the technique is being used to design window responses. The rectangular window is the only window to have the first and last impulses of half of the IR equal to one another, and this half IR is indeed symmetrical. Furthermore, the increasing values of the first-half IR coefficients of all windows ensure that the second problem is never encountered.

The algorithm lends itself well to computer implementation. It is possible to pass from the required intensity response in terms of extinction ratio and stopband in the frequency domain to the required window IR and number of samples. Therefore, using the algorithm described above it is possible to pass from the design specifications via the appropriate window to the design structure.

TABLE I

Window	k_2	k_3	k_4	k_5
Rectangular	0.2	0.25	0.3333	0.5
Triangular	0.3333	0.25	0.2	0.8333
Hanning	0.3515	0.269	0.1879	0.925
Hamming	0.3545	0.2551	0.114	0.924
Blackman	0.3514	0.2395	0.1227	0.9674

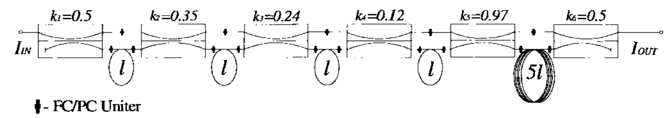

Fig. 3. The prototype structure showing the nominal coupling ratios used.

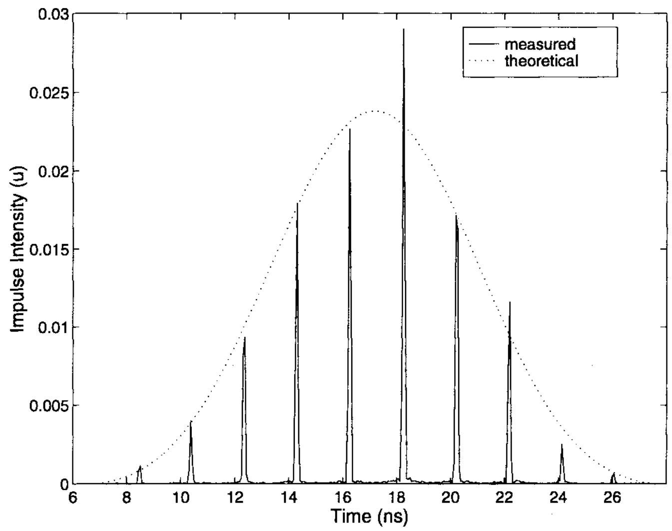

Fig. 4. Averaged time-domain response of the Mach–Zehnder lattice shown along with the ideal Blackman window shape.

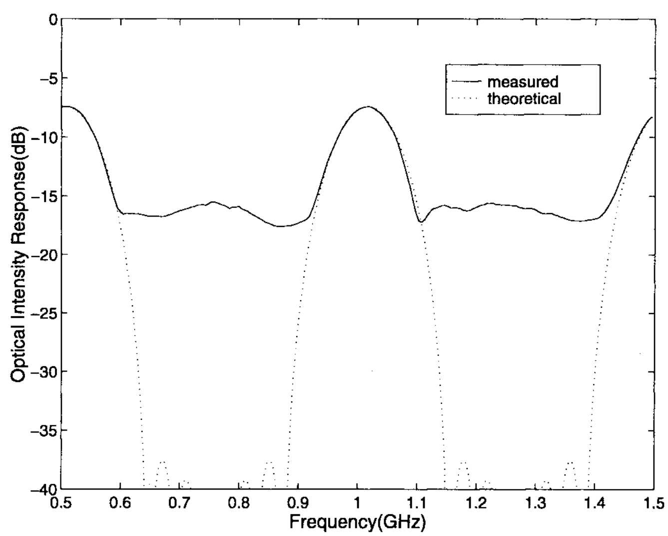

Fig. 5. Comparison of the simulated and average measured optical-intensity frequency response.

The values of coupling ratios of the central couplers for the best known windows for a six-coupler structure are shown in Table I where k_1 and k_6 are 0.5.

III. Experimental Results and Comments

A prototype structure was built using commercially available fiber-connector/physical-contact (FC/PC) connected fiber patch-chords and fused tapered couplers. The coupling ratios were chosen to be as close as possible to the ideal values for the six-coupler structure which generates a Blackman IR. The prototype structure is shown in Fig. 3 and was built using a unit delay length of 40 cm. This corresponds to a time delay of 2 ns, or a frequency response period of 500 MHz. Measurements were made using the Hewlett-Packard Lightwave Component Analyzer 8703A, with a Fabry–Perot source at 1300 nm, which can be intensity modulated by a microwave signal between 130 MHz and 20 GHz. The short coherence length of the source, estimated at around 300 μm, was significantly shorter not only than the structure unit delay, but also than the practical difference in nominally equal path lengths, making coherent optical interference unlikely. The measured response is shown in the time and frequency domains in Figs. 4 and 5, respectively. The time-domain measurements show the impulses follow the window coefficients closely for the first half of the window IR, but show notable discrepancies in the second half. The frequency-domain optical-intensity response, shown around the center frequency of 1 GHz, exhibits good agreement in the shape of the main lobe, but a poor extinction ratio compared to the expected value of approximately 27 dB (optical).

The shortfall in this response is believed to be due to differences of the values in the prototype from the design values. This is caused by three main factors. Firstly, there is the loss in the couplers and the connectors. If the losses in both arms of the structure were equal they would attenuate the response evenly and the extinction ratio and shape of the response would remain unchanged. However, in the case of the prototype, the loss is greater in the lower arm than in the upper, as there is approximately twice the number of connectors in the former as there is in the latter. The second prototype discrepancy is concerned with coupling ratios being inaccurate and a departure of the behavior of the couplers from the symmetrical model. This causes the stopband floor of the frequency response to be raised in a periodic way. The final source of error is due to inaccuracies in the delay lengths, which are hard to avoid when using connected components. These cause the response to shift slightly in the frequency domain, and again, raise the response floor. The effects become more pronounced as the frequency increases—unlike the periodic effects of coupler inaccuracies.

IV. Conclusion

The passive Mach–Zehnder lattice has been rigorously analyzed and the results used to create a synthesis algorithm for a modified Mach–Zehnder lattice which is capable of producing symmetrical responses, and in particular, window responses. The windows can be used to implement microwave-frequency filtering functions on intensity modulated light. The structure is attractive because it generates the window coefficients using only passive components rather than relying on optical amplification. A prototype structure has been built and tested, with results validating the theory. The results also show that the desired main lobe width is fairly insensitive to structure parameter errors, but that better accuracy is required to produce a good extinction ratio.

References

[1] B. Moslehi, J. W. Goodman, M. Tur, and H. J. Shaw, "Fiber optic lattice signal processing," *Proc. IEEE*, vol. 72, pp. 909–930, July 1984.

[2] K. P. Jackson, S. A. Newton, B. Moslehi, M. Tur, C. C. Cutler, J. W. Goodman, and H. J. Shaw, "Optical fiber delay line signal processing," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 193–209, Mar. 1985.

[3] L. N. Binh, N. Q. Ngo, and S. F. Luk, "Graphical representation and analysis of the Z-shaped double-coupler optical resonator," *J. Lightwave Technol.*, vol. 11, pp. 1782–1792, Nov. 1993.

[4] E. C. Heyde and R. A. Minasian, "A solution to the synthesis problem of recirculating optical delay line filters," *IEEE Photon. Technol. Lett.*, vol. 6, pp. 833–835, July 1994.

[5] J. Capmany, J. Cascón, J. L. Martín, and J. Martí, "Fiber-optic delay line filter synthesis using a modified Padé method," *Electron. Lett.*, vol. 31, no. 6, pp. 479–480, 1995.

[6] J. Capmany, J. Cascón, J. L. Martín, S. Sales, D. Pastor, and J. Martí, "Synthesis of fiber-optic delay line filters," *J. Lightwave Technol.*, vol. 13, pp. 2003–2012, Oct. 1995.

[7] C. C. Wang, "High-frequency narrow-band single-mode fiber-optic transversal filters," *J. Lightwave Technol.*, vol. LT-5, Jan. 1987.

[8] S. Sales, J. Capmany, J. Martí, and D. Pastor, "Solutions to the synthesis problem of optical delay line filters," *Opt. Lett.*, vol. 20, no. 23, pp. 2438–2440, 1995.

[9] A. V. Oppenheim and R. W. Schafer, *Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1985.

[10] F. J. Harris, "On the use of windows for harmonic analysis with the discrete Fourier transform," *Proc. IEEE*, vol. 66, pp. 51–82, Jan. 1978.

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